Vacuum Condensates and the Anomalous Magnetic Moment of a Dirac Fermion

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Abstract. We address anticipated fermion-antifermion and dimension-4 gauge-field vacuum-condensate contributions to the magnetic portion of the fermion-photon vertex function in the presence of a vacuum with nonperturbative content, such as that of QCD. We discuss how inclusion of such condensate contributions may lead to a *vanishing* anomalous magnetic moment, in which case vacuum condensates may account for the apparent consistency between constituent quark masses characterizing baryon magnetic moments and those characterizing baryon spectroscopy.

1 INTRODUCTION

One of the earliest successes of quantum electrodynamics (QED) was the successful calculation of the anomalous magnetic moment of an electron. [1,2] Such a calculation is, of course, performed within the context of a purely perturbative quantum field theory whose vacuum does not permit the formation of such nonperturbative exotica as electron-positron condensates and multiple-photon condensates. This is in contrast to the vacuum characterizing quantum chromodynamics (QCD), which clearly has nonperturbative content, as evident from the occurrence of quark-antiquark, multiple-gluon, and higher dimensional quark-gluon QCD-vacuum condensates which characterize QCD sum-rule applications. [3] Although QCD has had a great deal of success in predicting hadronic physics (particularly at high energies), it almost never is made to address the low-energy properties of its fundamental fermions (*i.e.*, quarks), even though analogous lepton properties are convincingly addressed by QED.

Part of the reason for this, of course, is confinement. We are completely incapable of measuring the magnetic moment of a quark in the same way that we can measure the quantity $(g-2)$ for a lepton. The strong coupling is itself ambiguous at low momenta. However, there does exist model-dependent but well-established and simple phenomenology that successfully determines the magnetic moments of baryons in terms of the magnetic moments of their constituent quarks. [4,5] The quark masses characterizing quark-magnetons within this phenomenology are suprisingly consistent with those characterizing hadron spectroscopy within the nonrelativistic quark model. [5–7] Thus, we do appear to have an indirect way of talking about quark magnetic moments, at least within the context of an hadronic environment, suggesting that there may be a value in having QCD address the QED-like question of what a quark's anomalous magnetic moment really is.

In Section 2, we review how the anomalous magnetic moment of a Dirac fermion is obtained in QED. The result $\mathcal{K}F_2(0) = e^2Q^2/8\pi^2$ (Q is the fermion charge) is obtained *entirely* from the single-photon-exchange vertex correction of Fig. 1. [2] If such a fermion were a quark, it would also couple to massless QCD gluons. An inevitable QCD modification of the perturbative calculation would entail the replacement of the momentum-k photon in Fig. 1 with gluons, including appropriate colour factors $\lambda^a_{ij}/2$ at each terminus of the gluon line G^a. The net effect of incorporating such gluon lines would be to augment the factor e^2Q^2 in $\mathcal{K}F_2(0)$ with an additional factor of $4g_s^2/3$ [4/3 is the $SU(3)_c$ group theoretical factor $\lambda^a_{ij}\lambda^a_{ji}/4$]. Since we can anticipate that $\alpha_s >> \alpha$ at low momenta, we would (naively) anticipate a quark anomalous magnetic moment approximately given by $\mathcal{K}F_2(0) \cong 2\alpha_s(0)/3\pi$. Such an anomalous moment is likely to be sufficiently large to compromise the success of the static quark model in calculating baryon magnetic moments. [4,5]

An immediate problem arises, of course, from attempting at all to utilize α_s in the soft-momentum (large-distance) limit. Mattingly and Stevenson have argued [8]

that α_s in this limit should approach an infrared-stable fixed point somewhat smaller than unity, a result corroborated by recent work [9] exploring the possible linkage between linear-sigma-model hadronic phenomenology and low-energy QCD. It has also been argued [10] that values of α_s near unity ($\alpha_s = 3/\pi$) induce chiral-symmetry breaking responsible for the low-energy transition from QCD to a chiral-lagrangian theory of mesons. Despite the arguments of ref. [8], there is contradictory evidence that an infrared-stable fixed point cannot occur unless n_f is substantially larger than three. [11−13] Arguments have also been advanced for the existence of an order-unity infrared attractor for the $n_f = 3$ QCD β-function that effectively corresponds to the (finite) coupling strength for the infrared boundary of QCD. The idea of introducing $\alpha_s(0)$ as a phenomenological parameter has support from other empirical contexts, as well. [14]

In any case, we see that there exists motivation for having the infrared limit of QCD characterized by an effective value of α_s near unity.[1] This suggests a large QCD contribution to the quark's anomalous magnetic moment, $\mathcal{K}F_2(0) \cong 2\alpha_s(0)/3\pi \cong 0.2$, arising *entirely* from purely perturbative single-gluon exchange. Such a 20% increase in the quark's magnetic moment has significant consequences when applied to constituent-quark-model estimates of nucleon (and other baryon) magnetic moments. The magneton characterizing such estimates is consistent with baryon constituent quark masses $m = m_d \cong 360\ MeV$. [7] Given empirical constraints, however, a 20% increase in the quark's true magnetic moment necessarily entails a concomitant 20% increase in the true constituent quark mass (m_{qk}) appearing in the quark magneton:

$$\mu_{empirical} = e^2/(2 \cdot 360\ MeV/c^2) = (e^2/2m_{qk})[1 + \mathcal{K}F_2(0)]\ . \tag{1.1}$$

Based on the above estimate, Eq. (1.1) suggests that the true constituent quark mass is elevated to 430 MeV/c^2, a value significantly larger than the constituent light quark mass characterizing hadron spectroscopy. Of course, there is latitude in this estimate (depending on $\alpha_s(0)$), but the net result is clearly a disequilibration of spectroscopic and magnetic-moment constituent-quark masses.

The above arguments suggest the need for a more careful attempt to ascertain the full QCD contribution to the anomalous magnetic moment of its fundamental-fermion constituents. Such an attempt must necessarily include incorporation of QCD-vacuum condensates which have already been introduced into field-theoretical calculations of QCD vacuum-polarization diagrams and the two-current correlation functions associated with QCD sum-rule applications. [3,15] The same field-theoretical techniques that enable the evaluation of vacuum condensate contribu-

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1) Refs [8] and [9] respectively advance values of 0.82 and 0.72 for α_s in the infrared region. Ref. [13] suggests an infrared attractor at $\alpha_s/\pi \cong 0.4$. This latter estimate is based upon [2|2] and [1|3] Padé approximants for the higher-than-one-loop terms in the $n_f = 3$ β-function, in conjunction with a large and negative asymptotic-error-formula estimate of that β-function's five loop term; *i.e.,* note that the value of $x_d (= \alpha_s/\pi)$ in Figs. 10 and 11 of ref. [13] is near 0.4 when $R_4 (\equiv \beta_4/\beta_0)$ is equal to -850, as predicted in ref. [12].

tions to two-point functions can be utilized to determine corresponding vacuum condensate contributions to QCD three-point functions as well.

In Section 3, we show how fermion-antifermion condensate contributions to the electromagnetic vertex function can be calculated, using sum-rule methods delineated in ref. [16]. For simplicity, we work within QED, but assume the presence of a chiral noninvariant vacuum capable of forming such a condensate. The startling result of this calculation is that the real part of the magnetic portion of the vertex correction vanishes over the soft momentum range $0 < q^2 < 4m_{qk}^2$, a result suggesting the absence of *any* fermion-antifermion condensate contribution to $\mathcal{K}F_2(0)$. However, this same magnetic term exhibits an imaginary part over the same kinematic region, in apparent correspondence (within a QCD context) to a dispersive contribution *below* the threshold for a "physical" quark-antiquark intermediate state.

In Section 4, we calculate the covariant-gauge dimension-4 gluon-condensate contributions to the electromagnetic vertex function. The use of a covariant gauge is shown to be necessary in order to retain the configuration-space translational invariance within the Feynman amplitude that is required for the factorization of external lines from the truncated vertex Green's function. Both the magnetic and the electric-charge contribution to the vertex correction are shown to be highly divergent on-shell. The naive interpretation of this result is to say that quarks exhibit an infinite magnetic moment, since the divergence of the magnetic contribution cannot be removed by renormalization, which has already been applied to ensure that the electric charge form-factor is unity at $q^2 = 0$. Obviously, particles with infinite magnetic moments can never appear to be free, consistent with confinement expectations (at least for quarks). However, a true demonstration of confinement on this basis would have to account for the full cancellation of such divergent magnetic-moment contributions within colour-singlet (only) constructions of quarks and gluons.

In Section 5, we attempt to interpret the salient features of the previous two sections. We argue that the kinematic extension of the dispersive domain found in Section 3 can be explained as a consequence of the Goldstone theorem – that the underlying assumptions of the calculation necessarily entail the existence of zero-mass Goldstone bosons as part of the physical particle spectrum. We then discuss the divergent gluon-condensate contributions to the electromagnetic vertex, and argue that such contributions must absorb two powers of the coupling to be renormalization-group invariant. This result suggests that these effects are, strictly speaking, a tree-order contribution, as opposed to a one-loop order contribution analogous to the purely perturbative case delineated in Section 2. If one then reformulates the perturbative series appropriately, the anomalous magnetic moment is seen to vanish entirely as a consequence of the differing degrees of divergence between magnetic and electric-charge contributions to the gluon-condensate component of the vertex correction. Such an interpretation, if correct, would account for the compatibility of the naive magnetic moment of constituent quarks with the constituent-quark masses that characterize hadron spectroscopy.

2 METHODOLOGY OF THE PURELY PERTURBATIVE CASE

Purely perturbative contributions to the fermion-antifermion-photon Green function G^μ, as depicted in Fig. 2, can be expressed in terms of time-ordered products of Heisenberg fields ψ^h, $\bar{\psi}^h$ and A_μ^h:

$$G_\mu(p_2,p_1) = \int d^4x' \int d^4y' < 0|T\psi^h(x')A_\mu^h(0)\bar{\psi}^h(y')|0 > e^{ip_2\cdot x'}e^{-ip_1\cdot y'}, \tag{2.1}$$

To one-loop order, the correction to this vertex is obtained in terms of interaction-picture fields ψ, $\bar{\psi}$, A_μ via the Wick-Dyson expansion of the time-ordered product within (2.1),

$$\begin{aligned} &< 0|T\psi^h(x')A_\mu^h(0)\bar{\psi}^h(y')|0 > \\ &\quad =< 0|T\psi(x')\, exp\left[-ieQ\int d^4w\ \bar{\psi}(w)\gamma^\sigma\psi(w)A_\sigma(w)\right]A_\mu(0)\bar{\psi}(y')|0 > \quad , \end{aligned} \tag{2.2}$$

in which case

$$\begin{aligned} G_\mu(p_2,p_1) = &\int d^4x' d^4y' e^{ip_2\cdot x'}e^{-ip_1\cdot y'} \\ &\Big\{(-ieQ)\int d^4w < 0|T\psi(x')\bar{\psi}(w)|0 > \gamma^\sigma < 0|T\psi(w)\bar{\psi}(y')|0 > \\ &\qquad \cdot < 0|TA_\sigma(w)A_\mu(0)|0 > \\ &+ (-ieQ)^3\int d^4x\ d^4y\ d^4z < 0|T\psi(x')\bar{\psi}(x)|0 > \gamma^\tau < 0|T\psi(x)\bar{\psi}(y)|0 > \\ &\qquad \cdot\gamma^\sigma < 0|T\psi(y)\bar{\psi}(z)|0 > \gamma^\rho < 0|T\psi(z)\bar{\psi}(y')|0 > \\ &\qquad \cdot < 0|TA_\tau(x)A_\rho(z)|0 >< 0|TA_\sigma(y)A_\mu(0)|0 > \\ &+\mathcal{O}[(ieQ)^5]\Big\} \end{aligned} \tag{2.3}$$

One obtains (2.3) by dropping all normal-ordered terms from the Wick-Dyson expansion of the time-ordered product on the right side of (2.2), as normal-ordering necessarily places annihilation operators strictly against the purely perturbative vacuum ket $|0>$:

$$< 0| : (Product\ of\ any\ number\ of\ field\ operators) : |0 >= 0 \tag{2.4}$$

Eq. (2.4) is no longer automatic if the vacuum is assumed to have nonperturbative content, notationally delineated by replacing $|0>$ with the nonperturbative (*i.e.,* QCD) vacuum $|\Omega>$. Indeed, such nonperturbative vacuum expectation values provide the explicit mechanism by which QCD-vacuum condensates contribute to field-theoretical current-correlation functions responsible for QCD sum-rules. [15] This same mechanism enables vacuum condensates to contribute to Feynman amplitudes in general.

Before we discuss such nonperturbative contributions to the fermion-antifermion-photon vertex of Fig. 2, it will prove methodologically useful for us to outline how the purely perturbative contribution is obtained. The two-field vacuum expectation values in (2.3) are simply configuration-space Feynman propagators,

$$< 0|T\psi(x)\bar{\psi}(y)|0 >= i\int \frac{d^4q}{(2\pi)^4}e^{-iq\cdot(x-y)}\frac{(\gamma\cdot q)+m}{q^2-m^2+i|\epsilon|}, \tag{2.5}$$

$$< 0|TA_\tau(x)A_\rho(z)|0 >= -i\int \frac{d^4k}{(2\pi)^4}e^{-ik\cdot(x-z)}\left(\frac{g_{\tau\rho}}{k^2+i|\epsilon|}-\frac{(1-\xi)k_\tau k_\rho}{k^4}\right), \tag{2.6}$$

where ξ is an (irrelevant) gauge-parameter coefficient of the longitudinal propagator component. The translational invariance of (2.5) and (2.6) enables factorization of external lines from the full one-loop contribution to G_μ:

$$\begin{aligned} G_\mu(p_2,p_1) = i\left(\frac{\not{p}_2+m}{{p_2}^2-m^2}\right)\left(\frac{-ig_{\mu\sigma}}{(p_1-p_2)^2}\right)[(-ieQ)\{\gamma^\sigma+\Gamma^\sigma(p_2,p_1)\}] \\ \cdot i\left(\frac{\not{p}_1+m}{{p_1}^2-m^2}\right), \end{aligned} \tag{2.7a}$$

$$\begin{aligned} &\Gamma^\sigma(p_2,p_1) \\ &= -\frac{ie^2Q^2}{(2\pi)^4}\int\frac{d^4k}{k^2}\gamma^\tau\left(\frac{\not{p}_2-\not{k}+m}{p_2-k^2-m^2}\right)\gamma^\sigma\left(\frac{\not{p}_1-\not{k}+m}{p_1-k^2-m^2}\right)\gamma_\tau+\mathcal{O}(e^4) \end{aligned} \tag{2.7b}$$

The truncated vertex function (2.7b) corresponds to the Feynman diagram in Fig. 1. The leading contribution to the anomalous magnetic moment of the fermion can be extracted from an explicitly finite coefficient $S(q^2)$ within this (unrenormalized) vertex function;

$$\Gamma^\sigma(p_2,p_1)\equiv e^2Q^2\left[R((p_2-p_1)^2)\gamma^\sigma+\frac{2S((p_2-p_1)^2)}{m}(p_1^\sigma+p_2^\sigma)\right] \tag{2.8}$$

where $S(0)$, as extracted from explicit evaluation of (2.7b) is found to be [2]

$$S(0)=-\frac{1}{32}\pi^2 \tag{2.9}$$

Making use of the Gordon decomposition relation

$$\bar{u}(p_2)(p_1^\sigma+p_2^\sigma)u(p_1)=\bar{u}(p_2)[2m\gamma^\sigma-i\sigma^{\sigma\mu}(p_2-p_1)_\mu]u(p_1) \tag{2.10}$$

one finds that the unrenormalized vertex function may be expressed in terms of the momentum transfer $q\equiv(p_2-p_1)$:

$$
\begin{aligned}
&\bar{u}(p_2)[\gamma^\tau + \Gamma^\sigma(p_2,p_1)]u(p_1) \\
&\quad = \bar{u}(p_2)\left[(1+e^2Q^2[R(q^2)+4S(q^2)])\gamma^\tau - \frac{2ie^2Q^2}{m}S(q^2)\sigma^{\tau\mu}q_\mu\right]u(p_1) \\
&\quad \equiv Z\bar{u}(p_2)\left[F_1(q^2)\gamma^\tau + \frac{i}{2m}\sigma^{\tau\mu}q_\mu \mathcal{K}F_2(q^2)\right]u(p_1) \qquad (2.11)
\end{aligned}
$$

where $F_1(q^2)$ is the fermion's charge form-factor and $\mathcal{K}F_2(0)$ is the fermion's anomalous magnetic moment. The requirement that $F_1(0) = 1$ imposes the following rescaling of the amplitude:

$$Z = 1 + e^2Q^2[(R(0) + 4S(0)] + \mathcal{O}(e^4) \qquad (2.12)$$

in which case the anomalous magnetic moment is given by

$$
\begin{aligned}
\mathcal{K}F_2(0) &= \frac{-4e^2Q^2S(0)}{1+e^2Q^2[R(0)+4S(0)]} \\
&= -4e^2Q^2S(0) + \mathcal{O}(e^4) \qquad (2.13)
\end{aligned}
$$

The finite calculated value of $S(0)$ [eq.(2.9)] ensures a finite anomalous magnetic moment: [1,2]

$$\mathcal{K}F_2(0) = \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2) \qquad (2.14)$$

By contrast, the coefficient $R(q^2)$ within (2.8) is divergent at $q^2 = 0$. Of course, additional self-energy and bremmstrahlung diagrams also contribute to the full vertex contribution, and these contributions are necessary to obtain a finite (and correct) answer for the renormalized charge-form-factor slope $F_1'(0)$. However, such contributions only enter the evaluation of $R(q^2)$ within the full unrenormalized vertex; $S(q^2)$ [and, consequently, $\mathcal{K}F_2(0)$] arises *entirely* from the Feynman amplitude (2.7b). In the two sections that follow we will respectively calculate the fermion-antifermion condensate and gluon-condensate contributions to $S(q^2)$, since $S(0)$ alone appears to determine the anomalous magnetic moment.

3 FERMION-ANTIFERMION CONDENSATE CONTRIBUTION TO THE ELECTROMAGNETIC VERTEX

The fermion-antifermion condensate contribution to the electromagnetic vertex of Fig. 2 has been evaluated at length. [17] We recapitulate the essential features of this calculation here. The Wick-Dyson expansion for the time-ordered product of fermion-antifermion fields is

$$T\psi(x)\bar{\psi}(y) = <0|T\psi(x)\bar{\psi}(y)|0> + :\psi(x)\bar{\psi}(y): \qquad (3.1)$$

If the true vacuum $|\Omega>$ does not exhibit chiral-invariance, the vacuum expectation value of (3.1) will acquire not only the perturbative configuration-space fermion propagator (2.5) but also a *nonperturbative propagator* component [16,18]

$$< \Omega|T\psi(x)\bar{\psi}(y)|\Omega> = <0|T\psi(x)\bar{\psi}(y)|0> + <\Omega| :\psi(x)\bar{\psi}(y): |\Omega> \tag{3.2a}$$

$$\begin{aligned} <\Omega| :\psi(x)\bar{\psi}(y): |\Omega> &= \int d^4k \; e^{-ik\cdot(x-y)}(\not{k}+m)\mathcal{F}(k); \\ \int d^4k \; \mathcal{F}(k)e^{-ik\cdot x} &\equiv <f\bar{f}> \frac{J_1(m\sqrt{x^2})}{6m^2\sqrt{x^2}} \end{aligned} \tag{3.2b}$$

In (3.2) $<f\bar{f}>$ is proportional (up to group theoretical factors) to the fermion-antifermion condensate; $<f\bar{f}> = -<\bar{q}q>$ for the case of $SU(3)_c$ fundamental-representation fermions - *i.e.,* quarks. The additional contribution to (3.2a) enters the Feynman amplitude for the vertex correction (as indicated symbolically in Fig. 3) by replacing perturbative fermion propagators within (2.3) with the appropriate nonperturbative propagator functions. This net extra fermion-antifermion contribution is given by

$$\begin{aligned} &\Delta G_\mu(p_2,p_1) \\ &= \int d^4x' d^4y' e^{ip_2\cdot x'} e^{-ip_1\cdot y'} \Big\{ (-ieQ)^3 \int d^4x \; d^4y \; d^4z < 0|T\psi(x')\bar{\psi}(x)|0> \gamma^\tau \\ &\quad \cdot \Big[<0|T\psi(x)\bar{\psi}(y)|0> \gamma^\sigma <\Omega| :\psi(y)\bar{\psi}(z): |\Omega> \\ &\qquad + <\Omega| :\psi(x)\bar{\psi}(y): |\Omega> \gamma^\sigma <0|T\psi(y)\bar{\psi}(z)|0> \Big] \gamma^\rho \\ &\quad \cdot <0|T\psi(z)\bar{\psi}(y')|0><0|TA_\tau(x)A_\rho(z)|0><0|TA_\sigma(y)A_\mu(0)|0> \Big\} \end{aligned} \tag{3.3}$$

[The term in which two nonperturbative vacuum expectation values appear vanishes. [17]]

The momentum-space realization of (3.3) is straightforward to obtain by use of (3.2b) in conjunction with (2.5) and (2.6). The explicit translational invariance of (3.2b) permits the same factorization of external lines from the truncated vertex as occurs in the purely perturbative case. The contribution of Fig. 3a, for example, is obtained from (2.7b) by replacing $[(k-p_2)^2 - m^2]^{-1}$ with $-i(2\pi)^4\mathcal{F}(k-p_2)$; similarly the contribution of Fig. 3b is obtained by replacing $[(k-p_1)^2 - m^2]^{-1}$ with $-i(2\pi)^4\mathcal{F}(k-p_1)$. The net effect of these changes is the following $<f\bar{f}>$-sensitive correction to the truncated electromagnetic vertex: [17]:

$$\begin{aligned} \Delta\Gamma^\mu(p_2,p_1) = -ie^2Q^2\{ &[-2\not{p}_1\gamma^\mu\not{p}_2 + 4m(p_1^\mu + p_2^\mu) - 2m^2\gamma^\mu] \; I \\ &+[2\gamma^\rho\gamma^\mu\not{p}_2 + 2\not{p}_1\gamma^\mu\gamma^\rho - 8mg^{\mu\rho}] \; I_\rho - 2\gamma^\rho\gamma^\mu\gamma^\sigma \; I_{\rho\sigma}\}, \end{aligned} \tag{3.4}$$

where the integrals $I, I_\rho, I_{\rho\sigma}$ (after trivial shifts in the integration variable) are

$$[I;\ I_\rho;\ I_{\rho\sigma}] = -i\int d^4k \frac{\mathcal{F}(k)[1;\ k_\rho + p_{1\rho};\ (k_\rho + p_{1\rho})(k_\sigma + p_{1\sigma})]}{(k+p_1)^2[(k+p_1-p_2)^2 - m^2]}$$
$$-i\int d^4k \frac{\mathcal{F}(k)[1;\ k_\rho + p_{2\rho};\ (k_\rho + p_{2\rho})(k_\sigma + p_{2\sigma})]}{(k+p_2)^2[(k+p_2-p_1)^2 - m^2]} \tag{3.5}$$

The aggregate contribution to $S(q^2)$ is the coefficient responsible for the anomalous magnetic moment within the vertex correction (2.8), and is found to be [17]

$$\Delta S(q^2) = \frac{m^2}{3}\int_0^1 dz \frac{1-z}{[m^2(1-z)^2 + q^2 z]^2}\left[\frac{< f\bar{f} > (1-z)}{6m}\right.$$
$$-[5m^2(1-z)^2 + q^2 z(1-4z)]\ R_2(p_1 z - p_2, mz)$$
$$\left.-[m^2 q^2 z(1-z)(3+5z) + q^4 z^2(1+2z)]\ R_3(p_1 z - p_2, mz)\right] \tag{3.6}$$

where for $p^2 > 0$

$$R_2(p,\ \mu) \equiv \int \frac{d^4k\ \mathcal{F}(k)}{(p-k)^2 - \mu^2 + i|\epsilon|} = \frac{< f\bar{f} >}{24m^3p^2}$$
$$\times \left[p^2 + m^2 - \mu^2 - \sqrt{[p^2 - (m-\mu)^2][p^2 - (m+\mu)^2]}\right], \tag{3.7}$$

$$R_3(p,\ \mu) \equiv \int \frac{d^4k\ \mathcal{F}(k)}{[(p-k)^2 - \mu^2 + i|\epsilon|]^2}$$
$$= -\frac{< f\bar{f} >}{24m^3p^2}\left[1 - \frac{p^2 + m^2 - \mu^2}{\sqrt{[p^2 - (m-\mu)^2][p^2 - (m+\mu)^2]}}\right]. \tag{3.8}$$

Explicit evaluation of the integral (3.6) on the $p_1^2 = p_2^2 = m^2$ mass shell in the region $0 < q^2 < 4m^2$ yields the following remarkable result:

$$
\begin{aligned}
Re[\Delta S(q^2)] = & \frac{m^2 < f\bar{f} >}{3} \int_0^1 dz \frac{1-z}{[m^2(1-z)^2+q^2z]^2} \\
& \cdot \left\{ \frac{1-z}{6m} - \frac{[5m^2(1-z)^2+q^2z(1-4z)][2m^2(1-z)+q^2z]}{24m^3[m^2(1-z)^2+q^2z]} \right. \\
& \left. + \frac{[m^2q^2z(1-z)(3+5z)+q^4z^2(1+2z)]}{24m^3[m^2(1-z)^2+q^2z]} \right\} \\
= & \, 0 \quad (0 < q^2 < 4m^2)
\end{aligned}
\tag{3.9}
$$

If $0 < q^2 < 4m^2$, the imaginary part of (3.6) is nonzero, despite the fact that the vertex momentum is *below* the kinematic threshold for the production of a quark-antiquark pair:

$$
\begin{aligned}
Im[\Delta S(q^2)] = & \frac{< f\bar{f} >}{72m} \int_0^1 dz \frac{1-z}{[m^2(1-z)^2+q^2z]^3} \\
& \cdot \left\{ [5m^2(1-z)^2+q^2z(1-4z)]z\sqrt{q^2(4m^2-q^2)} \right. \\
& \left. +[m^2q^2z(1-z)(3+5z)+q^4z^2(1+2z)] \frac{2m^2(1-z)+q^2z}{z\sqrt{q^2(4m^2-q^2)}} \right\} \\
= & \frac{< f\bar{f} >}{12m\sqrt{4m^2q^2-q^4}}.
\end{aligned}
\tag{3.10}
$$

The interpretation of these results will be deferred to Section 5. It should be noted here, however, that in the context of QCD, $e^2 < f\bar{f} > \to -g_s^2 < q\bar{q} > \sim m^3$, where m is understood to be the gauge invariant dynamical mass arising from the chiral noninvariance of the QCD vacuum. [19] Consequently, a single scale parameter m characterizes $g_s^2 \Delta S(q^2)$.

4 THE GLUON-CONDENSATE CONTRIBUTION TO THE ELECTROMAGNETIC VERTEX

The Wick-Dyson expansion for a time-ordered product of gluon fields includes both a configuration-space propagator and a normal-ordered piece:

$$T A_\tau(x) A_\rho(z) = < 0|T A_\tau(x) A_\rho(z)|0 > + : A_\tau(x) A_\rho(z) : \tag{4.1}$$

If the QCD vacuum $|\Omega >$ has nonperturbative content, the vacuum expectation value of (4.1) will acquire an additional nonperturbative component

$$< \Omega|T A_\tau(x) A_\rho(z)|\Omega > = < 0|T A_\tau(x) A_\rho(z)|0 > + < \Omega| : A_\tau(x) A_\rho(z) : |\Omega > \tag{4.2}$$

The first term on the right hand side of (4.2) is the purely perturbative configuration-space propagator given by (2.6). When evaluated in a translationally-invariant (*i.e.,* covariant) gauge, as required for the factorization of external lines from the truncated vertex function, the second term of (4.2) is given by [20]

$$< \Omega| : A_\tau(x) A_\rho(z) : |\Omega > = \frac{1}{8}\left[C(x-z)_\tau (x-z)_\rho + E g_{\tau\rho}(x-z)^2\right] \tag{4.3}$$

where the constants C and E are proportional to the dimension-4 gluon-condensate $< G^2 >$:

$$C = \frac{< G^2 >}{144} - \frac{< \Omega| : (\partial \cdot A)^2 : |\Omega >}{24} \tag{4.4a}$$

$$E = -\frac{5 < G^2 >}{288} - \frac{< \Omega| : (\partial \cdot A)^2 : |\Omega >}{48} \tag{4.4b}$$

and the gluon-condensate $< G^2 >$ is defined to be

$$< G^2 > = < 0| : G^a_{\mu\nu}(0) G^a_{\mu\nu}(0) : |0 > \tag{4.5}$$

with $G^a_{\mu\nu}$ the QCD field-strength tensor. The $< (\partial \cdot A)^2 >$ condensate is a reflection of the gauge dependence of (4.3); the cancellation of this condensate from several gauge-invariant amplitudes has been demonstrated in ref. [16].

The gluon-condensate contribution to the electromagnetic vertex is schematically represented by Fig. 4, and corresponds to replacing $< 0|T A_\tau(x) A_\rho(z)|0 >$ within (2.3) with the nonperturbative vacuum expectation value (4.3). Upon inclusion of $SU(3)_c$ vertex structure, the corresponding correction to the (untruncated) vertex Green's function is found to be

$$
\begin{aligned}
&\Delta G^\mu(p_2,p_1)\\
&= (-ieQ)(-ig_s)^2 Tr\left(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right)\int d^4x' d^4y' d^4x\ d^4y\ d^4z\ e^{ip_2\cdot y'}e^{-ip_1\cdot x'}\\
&\quad\cdot\left[i\int\frac{d^4q_1}{(2\pi)^4}\frac{\not{q}_1+m}{{q_1}^2-m^2}e^{-iq_1\cdot(y'-z)}\right]\gamma_\tau\left[i\int\frac{d^4q_2}{(2\pi)^4}\frac{\not{q}_2+m}{{q_2}^2-m^2}e^{-iq_2\cdot(z-y)}\right]\gamma_\sigma\\
&\quad\cdot\left[i\int\frac{d^4q_2}{(2\pi)^4}\frac{\not{q}_3+m}{{q_3}^2-m^2}e^{-iq_3\cdot(y-x)}\right]\gamma_\rho\left[i\int\frac{d^4q_4}{(2\pi)^4}\frac{\not{q}_4+m}{{q_4}^2-m^2}e^{-iq_4\cdot(x-x')}\right]\\
&\quad\cdot\frac{1}{8}\left[C(x-z)^\rho(x-z)^\tau+g^{\rho\tau}E(x-z)^2\right]\left[(-ig^{\mu\sigma})\int\frac{d^4k'}{(2\pi)^4}\frac{1}{k'^2}e^{ik'\cdot y}\right]\\
&= \frac{i^2}{8}(-ieQ)(-ig_s)^2\left(\frac{1}{3}\sum_a\frac{\delta^{aa}}{2}\right)\frac{1}{(2\pi)^{20}}\int d^4k' d^4q_1\ d^4q_2\ d^4q_3\ d^4q_4\\
&\quad\cdot\left[i\frac{\not{q}_1+m}{q_1^2-m^2}\right]\gamma_\tau\frac{\not{q}_2+m}{q_2^2-m^2}\gamma^\mu\frac{\not{q}_3+m}{q_3^2-m^2}\gamma_\rho\left[i\frac{\not{q}_4+m}{q_4^2-m^2}\right]\left[\frac{-i}{k'^2}\right]\\
&\quad\cdot\left[\int d^4x' e^{ix'\cdot(q_4-p_1)}\right]\left[\int d^4y' e^{iy'\cdot(p_2-q_1)}\right]\left[\int d^4y e^{iy\cdot(q_2-q_3+k')}\right]\\
&\quad\cdot\int d^4x\ d^4z\ e^{ix\cdot(q_3-q_4)}e^{iz\cdot(q_1-q_2)}\left[C(x-z)^\tau(x-z)^\rho+g^{\tau\rho}E(x-z)^2\right]\\
&= \frac{i^2}{8}(-ieQ)(-ig_s)^2\left(\frac{4}{3}\right)\frac{1}{(2\pi)^8}\left[i\frac{\not{p}_2+m}{p_2^2-m^2}\right]\int d^4q_2\ d^4q_3\\
&\quad\cdot\gamma_\tau\left\{\frac{\not{q}_2+m}{q_2^2-m^2}\gamma^\mu\frac{\not{q}_3+m}{q_3^2-m^2}\right\}\gamma_\rho\left[\frac{-i}{(q_3-q_2)^2}\right]\int d^4x\ d^4z\ e^{ix\cdot(q_3-p_1)}\ \cdot\\
&\quad\cdot e^{iz\cdot(p_2-q_2)}\left[C(x-z)^\tau(x-z)^\rho+g^{\tau\rho}E(x-z)^2\right]\left[i\frac{\not{p}_1+m}{p_1^2-m^2}\right]\\
&\equiv \frac{i^2}{8}(-ieQ)(-ig_s)^2\left(\frac{4}{3}\right)\frac{1}{(2\pi)^8}\left[i\frac{\not{p}_2+m}{p_2^2-m^2}\right]I^\mu(p_1,p_2)\left[i\frac{\not{p}_1+m}{p_1^2-m^2}\right].
\end{aligned}
\tag{4.6}
$$

To evaluate I^μ, as defined in the final line of (4.6), we make the following change of variables,

$$
\begin{gathered}
\mathcal{X}\equiv z-x;\mathcal{Z}\equiv x+z\\
x=\frac{1}{2}(\mathcal{Z}-\mathcal{X});z=\frac{1}{2}(\mathcal{X}+\mathcal{Z})
\end{gathered}
\tag{4.7}
$$

and we define

$$
H^\mu(q_2,q_3)\equiv\left\{\frac{\not{q}_2+m}{q_2^2-m^2}\gamma^\mu\frac{\not{q}_3+m}{q_3^2-m^2}\right\}
\tag{4.8}
$$

$$
\mathcal{P}\equiv\frac{1}{2}(p_1+p_2);\ \mathcal{Q}\equiv\frac{1}{2}(p_2-p_1)
\tag{4.9}
$$

We then find that

$$
\begin{aligned}
I^\mu(p_1,p_2) &\equiv \int d^4q_2\ d^4q_3\ \gamma_\tau H^\mu(q_2,q_3)\gamma_\rho \left[\frac{-i}{(q_3-q_2)^2}\right] \int d^4x\ d^4z\ e^{ix\cdot(q_3-p_1)} e^{iz\cdot(p_2-q_2)} \\
&\quad \cdot \left[C(x-z)^\tau(x-z)^\rho + g^{\tau\rho}E(x-z)^2\right] \\
&= (\frac{1}{2})^4 \int d^4q_2\ d^4q_3\ \gamma_\tau H^\mu(q_2,q_3)\gamma_\rho \left[\frac{-i}{(q_3-q_2)^2}\right] \int d^4\mathcal{X} d^4\mathcal{Z} e^{i\mathcal{X}\cdot[\mathcal{P}-\frac{1}{2}(q_2+q_3)]} \\
&\quad \cdot e^{i\mathcal{Z}\cdot[\mathcal{Q}-\frac{1}{2}(q_2-q_3)]} \left[C\mathcal{X}^\tau\mathcal{X}^\rho + g^{\tau\rho}E\mathcal{X}^2\right] \\
&= (\frac{1}{2})^4 \int d^4q_2\ d^4q_3\ \gamma_\tau H^\mu(q_2,q_3)\gamma_\rho \left[\frac{-i}{(q_3-q_2)^2}\right] \left\{Cg^{\alpha\rho}g^{\beta\tau} + Eg^{\rho\tau}g^{\alpha\beta}\right\} \\
&\quad \cdot (2\pi)^4\delta^4\left[\mathcal{Q}-\frac{1}{2}(q_2-q_3)\right] \int d^4\mathcal{X}\mathcal{X}_\alpha\mathcal{X}_\beta e^{i\mathcal{X}\cdot[\mathcal{P}-\frac{1}{2}(q_2+q_3)]} \\
&= (\frac{1}{2})^4 \int d^4q_2\ d^4q_3\ \gamma_\tau H^\mu(q_2,q_3)\gamma_\rho \left[\frac{-i}{(q_3-q_2)^2}\right] \left\{Cg^{\alpha\rho}g^{\beta\tau} + Eg^{\rho\tau}g^{\alpha\beta}\right\} \\
&\quad \cdot (2\pi)^4\delta^4\left[\mathcal{Q}-\frac{1}{2}(q_2-q_3)\right] \left(-\frac{\partial^2}{\partial\mathcal{P}^\alpha\partial\mathcal{P}^\beta}\right)(2\pi)^4\delta^4\left[\mathcal{P}-\frac{1}{2}(q_2+q_3)\right] \\
&= (\frac{1}{2})^4(2\pi)^8\left\{Cg^{\alpha\rho}g^{\beta\tau} + Eg^{\rho\tau}g^{\alpha\beta}\right\}\left(-\frac{\partial^2}{\partial\mathcal{P}^\alpha\partial\mathcal{P}^\beta}\right)\gamma_\tau H_\sigma(\mathcal{P}+\mathcal{Q},\mathcal{P}-\mathcal{Q}) \\
&\quad \cdot \gamma_\rho\left[\frac{-ig^{\mu\sigma}}{4\mathcal{Q}^2}\right]. \qquad (4.10)
\end{aligned}
$$

Factorization of the external photon in the final line of (4.10), in conjunction with factorization of the external fermions in the final line of (4.6), allows a clear determination of the gluon-condensate contribution to the truncated vertex function:

$$
\begin{aligned}
\Delta\Gamma^\sigma(p_2,p_1) &= \frac{i^2}{8}(-ig_s)^2\left(\frac{4}{3}\right)(\frac{1}{2})^4\frac{1}{4}\left[Cg^{\alpha\rho}g^{\beta\tau} + Eg^{\rho\tau}g^{\alpha\beta}\right]\gamma_\tau\left(-\frac{\partial^2}{\partial\mathcal{P}^\alpha\partial\mathcal{P}^\beta}\right) \\
&\quad \cdot\left\{\frac{\not{\mathcal{P}}+\not{\mathcal{Q}}+m}{(\mathcal{P}+\mathcal{Q})^2-m^2}\gamma^\sigma\frac{\not{\mathcal{P}}-\not{\mathcal{Q}}+m}{(\mathcal{P}-\mathcal{Q})^2-m^2}\right\}\gamma_\rho. \qquad (4.11)
\end{aligned}
$$

Define

$$
D_1 \equiv (\mathcal{P}-\mathcal{Q})^2 - m^2 = p_1^2 - m^2\ ;\ D_2 \equiv (\mathcal{P}+\mathcal{Q})^2 - m^2 = p_2^2 - m^2. \qquad (4.12)
$$

The partial derivatives within (4.11) are then found to be

$$
\begin{aligned}
&\frac{\partial^2}{\partial\mathcal{P}^\alpha\partial\mathcal{P}^\beta}\left\{\frac{1}{D_1D_2}(\not{\mathcal{P}}+\not{\mathcal{Q}}+m)\gamma^\sigma(\not{\mathcal{P}}-\not{\mathcal{Q}}+m)\right\} \\
&\quad = \left[B_{\alpha\beta}(\not{p}_2+m)\gamma^\sigma(\not{p}_1+m) + \frac{1}{D_1D_2}(\gamma_\beta\gamma^\sigma\gamma_\alpha + \gamma_\alpha\gamma^\sigma\gamma_\beta)\right], \qquad (4.13)
\end{aligned}
$$

where

$$A^\alpha \equiv \frac{\partial}{\partial P_\alpha}\frac{1}{D_1 D_2} = -\frac{2}{D_1^2 D_2^2}(p_1^\alpha D_2 + p_2^\alpha D_1) \tag{4.14a}$$

$$B^{\alpha\beta} \equiv \frac{\partial}{\partial P_\alpha} A^\beta = 2 D_1 D_2 A^\alpha A^\beta - \frac{2}{D_1^2 D_2^2}\left[g^{\alpha\beta}(D_1 + D_2) + 2(p_1^\alpha p_2^\beta + p_2^\alpha p_1^\beta)\right] \tag{4.14b}$$

Upon substitution of (4.13) into (4.11), we find that

$$
\begin{aligned}
\Delta\Gamma^\sigma(p_2,p_1) = & \frac{-g_s^2}{3\cdot 2^7}\left[Cg^{\alpha\rho}g^{\beta\tau}+Eg^{\rho\tau}g^{\alpha\beta}\right] \\
& \cdot\gamma_\tau\left[B_{\alpha\beta}(\not p_2+m)\gamma^\sigma(\not p_1+m)+\frac{1}{D_1D_2}(\gamma_\beta\gamma^\sigma\gamma_\alpha+\gamma_\alpha\gamma^\sigma\gamma_\beta)\right]\gamma_\rho \\
= & \frac{-g_s^2}{3\cdot 2^7}\Big\{C\Big[B_{\alpha\beta}\gamma^\beta(\not p_2+m)\gamma^\sigma(\not p_1+m)\gamma^\alpha \\
& \qquad +\frac{1}{D_1D_2}(\gamma^\beta\gamma_\beta\gamma^\sigma\gamma_\alpha\gamma^\alpha+\gamma^\beta\gamma_\alpha\gamma^\sigma\gamma_\beta\gamma^\alpha)\Big]+ \\
& \quad E\Big[g^{\alpha\beta}B_{\alpha\beta}\gamma_\tau(\not p_2+m)\gamma^\sigma(\not p_1+m)\gamma^\tau \\
& \qquad +\frac{1}{D_1D_2}(\gamma_\tau\gamma^\alpha\gamma^\sigma\gamma_\alpha\gamma^\tau+\gamma_\tau\gamma_\alpha\gamma^\sigma\gamma^\alpha\gamma^\tau)\Big]\Big\}
\end{aligned} \tag{4.15}
$$

We can then sandwich $\Delta\Gamma$ between on-shell spinors,

$$
\begin{aligned}
&\overline{u}(p_2)(\not p_2-m)=0, \\
&(\not p_1-m)u(p_1)=0,
\end{aligned} \tag{4.16}
$$

and utilize the relations

$$
\begin{aligned}
\frac{(\not p_1-m)}{p_1^2-m^2}u(p_1) &= \frac{1}{2m}u(p_1) \\
\overline{u}(p_2)\frac{(\not p_2-m)}{p_2^2-m^2} &= \overline{u}(p_2)\frac{1}{2m}
\end{aligned} \tag{4.17}
$$

Care must be taken not to cancel terms in the numerator resulting from naively applying the on-shell relations without regard for vanishing denominator factors of D_1 and D_2 in (4.15). After a fair amount of Dirac algebra, the cautious application of these identities [without yet setting $(p_1^2=p_2^2=m^2)$] to (4.15) taken between on-shell spinors yields the following result for the gluon-condensate contribution to the truncated electromagnetic vertex function $[D_1=D_2\equiv\mathcal{D}]$:

$$
\begin{aligned}
&\overline{u}(p_2)\Delta\Gamma^\sigma(p_2,p_1)u(p_1) \\
&= \frac{-g_s^2}{3\cdot 2^7}\overline{u}(p_2)\left\{\left[\frac{20C+8E}{\mathcal{D}^2} + \frac{8}{\mathcal{D}^3}\left(\frac{2E(2m^2+p_1\cdot p_2)}{\mathcal{D}} - C\right)\left(2p_1\cdot p_2 + \mathcal{D} + \frac{\mathcal{D}^2}{4m^2}\right)\right.\right. \\
&\quad \left. + \frac{8C}{\mathcal{D}^4}\left(2m^4 + 2(p_1\cdot p_2)^2 + \left(4m^2 + 7\mathcal{D} + \frac{\mathcal{D}^2}{4m^2}\right)p_1\cdot p_2 + 3\mathcal{D}^2 + 4m^2\mathcal{D}\right)\right]\gamma^\sigma \\
&\quad + (p_1^\sigma + p_2^\sigma)\left[-\frac{8}{m\mathcal{D}^2}\left(\frac{2E(2m^2+p_1\cdot p_2)}{\mathcal{D}} - C\right) - \frac{8C}{\mathcal{D}^3}\left(\frac{3\mathcal{D}}{2m} + 2m\right)\right. \\
&\quad \left.\left. - \frac{4C}{\mathcal{D}^3}\left(\frac{2p_1\cdot p_2}{m} - \frac{\mathcal{D}}{2m}\right)\right]\right\}u(p_1) \qquad (4.18)
\end{aligned}
$$

We now utilize the decomposition (2.8) to define the gluon-condensate contributions to $R(q^2)$ and $S(q^2)$ at $q^2 \to 0$. Since $\mathcal{D} \to 0$ on-shell, such contributions are highly divergent:

$$
[R(0)]_{<G^2>} = \frac{-g_s^2}{3\cdot 2^7 e^2 Q^2}\left[\frac{(96E+64C)m^4}{\mathcal{D}^4} + \mathcal{O}\left(\frac{m^2}{\mathcal{D}^3}\right)\right] \qquad (4.19)
$$

$$
[S(0)]_{<G^2>} = \frac{-g_s^2}{3\cdot 2^7 e^2 Q^2}\left[-\frac{(24E+12C)m^2}{\mathcal{D}^3} - \frac{C}{\mathcal{D}^2}\right] \qquad (4.20)
$$

It is evident from substitution of (4.20) into (2.13) that the gluon-condensate contribution to the anomalous magnetic moment of a quark appears to be divergent, a consequence of the fact that $S(0)$ is now divergent, as opposed to the purely perturbative case (Section 2) where $S(0)$ is manifestly finite. Moreover, the problem is not remedied by any cancellation between $< G^2 >$ and $< (\partial_\mu A^\mu)^2 >$ terms contributing to C and E [e.g. (4.4)], since no such cancellation can simultaneously render $(24E + 12C) = 0$ to remove the $\mathcal{D}^{-3}$ divergence and $C = 0$ to remove the $\mathcal{D}^{-2}$ divergence.

We note, of course, that finite quark magnetic moments appear to characterize constituent quarks in a number of phenomenologically successful applications, as discussed in Section 1. In Section 5, we will explore whether (2.13) is indeed applicable to the anomalous magnetic moment of a condensing fermion.

5 DISCUSSION

Quark Condensate Contributions

Let us begin by considering *in isolation* the fermion-antifermion condensate contribution to $S(q^2)$. If we disregard for now the nonzero imaginary part (3.10), a dispersive contribution necessarily indicative of physical intermediate states, we see from (3.9) that the fermion-antifermion condensate contribution to $\mathcal{K}F_2(0)$ is zero, as the real part of $S(0)$ is zero. In and of itself, this result decouples any quark-antiquark condensate effects from the quark's anomalous magnetic moment. As discussed in Section 1, however, purely perturbative effects are expected to give a substantial contribution to the quark's anomalous magnetic moment that would create a discrepancy between constituent-quark masses characterizing baryon spectroscopy and those characterizing baryon magnetic moments. The absence of any offsetting condensate contribution, however interesting in itself, leaves us with these phenomenological issues unresolved.

The occurence in $S(q^2)$ of a nonzero imaginary part when $0 < q^2 < 4m^2$ is also of interest. The purely perturbative contribution to $S(q^2)$ acquires an imaginary part only when $q^2 > 4m^2$, corresponding to being above the kinematic threshold for the production of a physical fermion-antifermion pair. In other words, the Feynman amplitude for the vertex correction acquires a dispersive component associated with the presence of a physical fermion-antifermion intermediate state. In the presence of a vacuum $|\Omega >$ that permits $< f\bar{f} >$ condensation (*i.e.,* the QCD vacuum), the kinematic threshold for this dispersive component is lowered from $q^2 = 4m^2$ to $q^2 = 0$.

To understand why this occurs, it may be useful to recapitulate the input assumptions of the calculation presented in Section 3. A single dynamical mass is assumed to characterize both the perturbative (2.5) and the nonperturbative (3.2b) fermion propagator, the latter being a reflection of the chiral noninvariance of the vacuum $|\Omega >$. Any attempt to differentiate between perturbative and nonperturbative propagator masses compromises the gauge invariance of $\mathcal{K}F_2(0)$, unless the Feynman rules utilized in obtaining $S(q^2)$ are themselves modified by 1PI contributions necessary to retain consistency with BRST identities. [21] We have chosen to utilize a single dynamical mass to characterize the Feynman amplitude in order to avoid these complications. Indeed, a similar need to equilibrate nonperturbative and perturbative propagator masses in order to maintain gauge invariance has been demonstrated within the context of condensate contributions to two-point functions, [22] and such two-point functions are also seen to acquire dispersive contributions for values of momentum-squared between zero and $4m^2$. [23] In any case, the mass we are using does not arise because of a lagrangian mass term, but rather as a consequence of the chiral nonivariance of the vacuum $|\Omega >$. Such violation of explicit chiral symmetry necessarily entails the presence of a massless Goldstone boson in the particle spectrum, in which case the extension of the dispersive component's kinematical domain from $q^2 > 4m^2$ to $q^2 > 0$ must be understood as

corresponding to the production of "physical" zero-mass Goldstone bosons. Within a QCD context, such Goldstone bosons are, of course, pions. We see, therefore, that the lowering of the threshold for $S(q^2)$'s dispersive component from $4m^2$ to zero may be a direct indication of QCD's transition from a gauge theory of quarks and gluons to a theory of low-energy hadron physics.

Gluon-Condensate Contributions

We have seen in Section 4 that the gluon-condensate contributions to $S(q^2)$ are divergent on-shell, leading via (2.13) to a divergent anomalous magnetic moment. It is worth examining whether the set of assumptions leading to (2.13) remains applicable when such nonperturbative contributions are present.

We first note that on the $\mathcal{D} = 0$ mass shell, the gluon-condensate contributions to $R(q^2)$ are even more divergent than the gluon-condensate contributions to $S(q^2)$; the former diverge like $1/\mathcal{D}^4$ (4.19) whereas the latter diverge like $1/\mathcal{D}^3$ (4.20). To leading order in α_s, there also exist gluon-condensate contributions to $R(q^2)$ arising from self-energy insertions (Fig. 5). However, gluon-condensate contributions to self-energies [16]

$$\Sigma(p) = \frac{\pi m^3 \alpha_s}{3(p^2 - m^2)} < \Omega| : (\partial \cdot A)^2 : |\Omega > + \frac{\pi \alpha_s [(p^2 - 3m^2) \not{p} + 3m^3]}{9(p^2 - m^2)^3} < G^2 > \quad , \tag{5.1}$$

when inserted into the Fig. 5 graphs, fail to cancel the leading $1/\mathcal{D}^4$ divergence of $R(q^2)$ arising from Fig. 4.

We note that QCD vacuum condensates are supposed to be renormalization-group (RG) invariant structures. Indeed, the RG-invariant dimension-4 gluon-condensate is not $< G^2 >$, but rather is $< \beta(\alpha_s) G^2/\alpha_s >$. To leading order in α_s, we note that $\beta(\alpha_s)/\alpha_s$ is proportional to α_s itself, in which case the RG-invariant condensate absorbs two factors of the strong coupling g_s; *i.e.,* if (4.19) and (4.20) are to be expressed in terms of RG invariants, then

$$g_s^2 < G^2 > \longrightarrow 4\pi < \alpha_s G^2 > \quad . \tag{5.2}$$

The condensate $< \alpha_s G^2 >$ is itself familiar to QCD sum-rule applications, [3] and has a sum-rule estimated magnitude ($0.045\ GeV^4$) somewhat larger than $(\Lambda_{QCD})^4$, indicative of a dimensionally-appropriate QCD momentum-scaling that is not subject to additional perturbative suppression. This suggests that (4.19) and (4.20) are in fact *order-unity* in the perturbation series. If one considers purely perturbative and gluon-condensate contributions together with other (perturbatively suppressed) contributions, one will then find for the truncated vertex $\bar{u}(p_2)[\gamma^\tau + \Gamma^\tau(p_2, p_1)]u(p_1)$, analogous to (2.8), that

$$\Gamma^\tau(p_2, p_1) = \gamma^\tau [[R(q^2)]_{<G^2>} + \mathcal{O}(g_s^2, e^2)] + \frac{2(p_1^\tau + p_2^\tau)}{m} [[S(q^2)]_{<G^2>} + \mathcal{O}(g_s^2, e^2)] \tag{5.3}$$

where, from (4.19,4.20) and (5.2),

$$[R(0)]_{<G^2>} \sim <\alpha_s G^2> m^4/\mathcal{D}^4 \ , \tag{5.4.a}$$
$$[S(0)]_{<G^2>} \sim <\alpha_s G^2> m^2/\mathcal{D}^3 \ . \tag{5.4.b}$$

Eq. (5.3) differs from (2.8), the definition of $R(q^2)$ and $S(q^2)$ for the purely perturbative case, only by omission of an overall factor of e^2Q^2, indicative that the leading vertex correction is no longer one order of perturbation theory removed from unity. As a consequence of this change, we now identify (5.3) with the final line of (2.11). Upon application of the renormalization condition requiring $F_1(0)$ to be constrained to unity, Eq. (2.13) is then replaced with the following relation:

$$\begin{aligned}\mathcal{K}F_2(0) &= \lim_{q^2\to 0} \frac{-4S(q^2)}{1+R(q^2)+4S(q^2)} \\ &= \lim_{\mathcal{D}\to 0} \frac{-4[S]_{<G^2>} + \mathcal{O}(g_s^2, e^2)}{1+[R]_{<G^2>}+4[S]_{<G^2>} + \mathcal{O}(g_s^2, e^2)} \\ &= 0. \end{aligned} \tag{5.5}$$

The last line follows trivially from (5.4). It should be noted that (5.5) is not just a statement about the gluon-condensate contribution, but rather a statement of what happens to all contributions to the anomalous magnetic moment [*i.e.,* the $\mathcal{O}(g_s^2, e^2)$ contributions] in the presence of a gluon-condensate contribution that is not suppressed perturbatively. All other condensate contributions, like the quark condensate contribution of Section 3, are expected to be suppressed by at least one factor of g_s^2.

Thus, the presence of a QCD vacuum $|\Omega>$ that permits gluon condensation appears to preclude the possibility of quarks acquiring any anomalous magnetic moment at all. This would suggest that quarks behave like naive Dirac fermions with magneton $eQ/2m$, where m is of order 350 MeV [recall that m has already been identified as a dynamical mass]. The idea that constituent quarks act like fundamental Dirac fermions is supported in other contexts as well. [24] Indeed, the absence of an anomalous magnetic moment for constituent quarks is of some phenomenological utility, in that it obviates the discrepancy anticipated on purely perturbative grounds (Section 1) between constituent quark masses characterizing baryon spectroscopy and those characterizing baryon magnetic moments.

Of course, there are necessarily *caveats* to all this – salient among these is the use of a strong coupling at $q^2 = 0$, as discussed in Section 1. The use of condensates obtained in a covariant gauge is also a nonstandard technique which, in principle, includes the presence of a gauge noninvariant condensate $<(\partial \cdot A)^2>$. This quantity can be shown to cancel algebraically from some other physical amplitudes, [16] and it is somewhat disconcerting to have it not do so here [except in the sense that $<(\partial \cdot A)^2>$, along with both $<\alpha_s G^2>$ and $<q\bar{q}>$, appears ultimately to be decoupled from $\mathcal{K}F_2(0)$]. However, we believe there is at least methodological merit

in the field theoretical calculation of condensate contributions to a vertex function. The techniques now exist to do such calculations; the challenge remaining is to make sense of the results.

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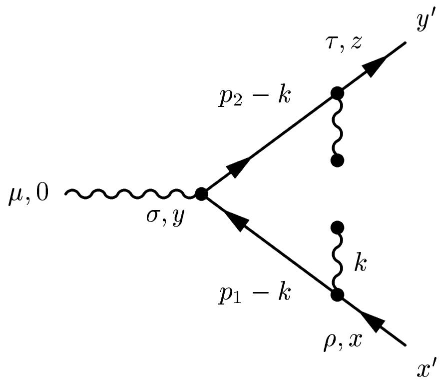

Fig. 4: The leading gluon-condensate contribution to the configuration-space fermion-antifermion-photon Green function.

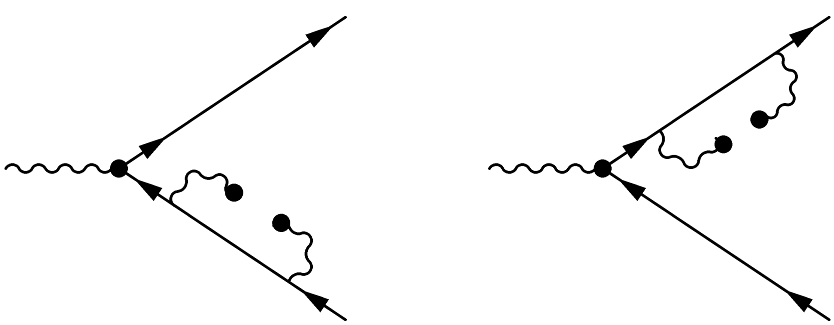

Fig. 5: Gluon-condensate contributions to quark self-energy corrections to an external electromagnetic field.

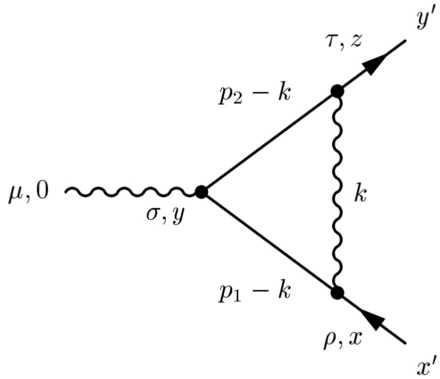

Fig. 1: The leading-order purely-perturbative electromagnetic vertex correction to the scattering of a fermion by an external electromagnetic field.

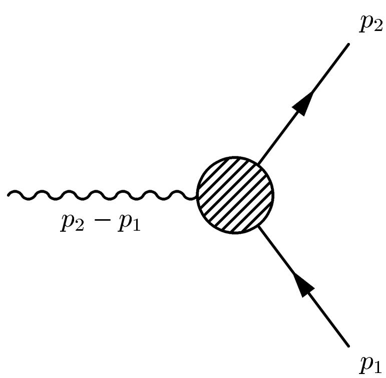

Fig. 2: The fermion-antifermion-photon Green function.

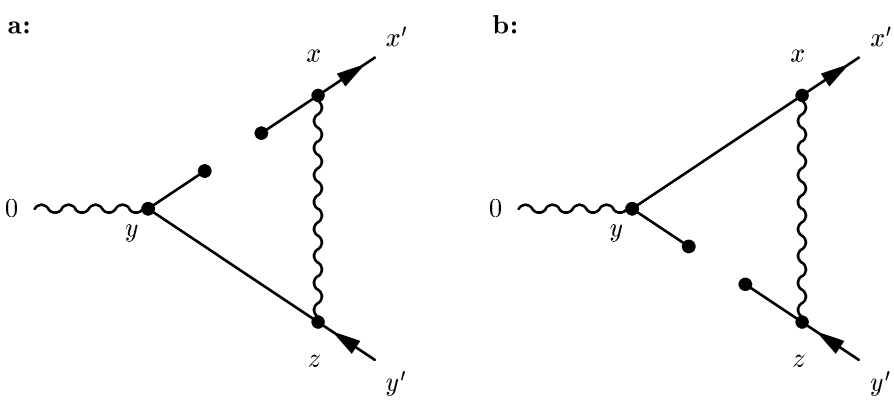

Fig. 3: The leading fermion-antifermion condensate contributions to the configuration-space fermion-antifermion-photon Green function.